

A Fourth Look At F#

Type Definitions

- Predefined, but not primitive in F#:

```
type bool = true | false;;
```

- Type constructor for lists:

```
type 'element list = List.empty |  
  :: of 'element * 'element list
```

Outline

- Enumerations
- Data constructors with parameters
- Type constructors with parameters
- Recursively defined type constructors

Programmer Defined Types

- New types can be defined using the keyword **type**
- These declarations define both:
 - *type constructors* for making new (possibly polymorphic) types
 - *data constructors* for making values of those new types

Example

```
type day = Mon | Tue | Wed | Thu | Fri | Sat | Sun;;  
type day = Fri | Mon | Sat | Sun | Thu | Tue | Wed  
  
let isWeekDay x = not (x = Sat || x = Sun) ;;  
val isWeekDay = fn : day -> bool  
  
isWeekDay Mon;;  
val it = true : bool  
  
isWeekDay Sat;;  
val it = false : bool
```

- **day** is the new *type constructor* and **Mon**, **Tue**, etc. are the new *data constructors*
- Why “constructors”? In a moment we will see how both can have parameters...

No Parameters

```
type day = Mon | Tue | Wed | Thu | Fri | Sat | Sun;;  
type day = Mon | Tue | Wed | Thu | Fri | Sat | Sun
```

- The *type constructor* **day** takes no parameters: it is not polymorphic, there is only one **day** type
- The *data constructors* **Mon**, **Tue**, etc. take no parameters: they are constant values of the **day** type
- Capitalize the names of data constructors

Strict Typing

```
type flip = Heads | Tails;;  
  type flip = Heads | Tails  
let isHeads x = (x = Heads);;  
  val isHeads = fn : flip -> bool  
isHeads Tails;;  
  val it = false : bool  
isHeads Mon;;  
error FS0001: This expression was expected to have  
type  
    flip  
but here has type  
    day  
enum flip {Heads, Tails};
```

- F# strictly enforces operations on new types
- Unlike C **enum**, no implementation details are exposed to the programmer. The following defines type *flip* with **Heads=0**, **Tails=1**.

Data Constructors In Patterns

```
let isWeekday d =  
    match d with  
        | Sat | Sun -> false  
        | _ -> true;;
```

- *Data constructors* can be used in patterns
- In this simple case, similar to constants
- More general case where constructor contains a typed data item, considered next

Outline

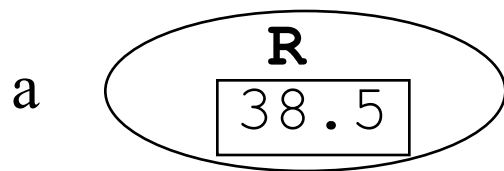
- Enumerations
- **Data constructors with parameters**
- Type constructors with parameters
- Recursively defined type constructors
- Farewell to F#

Datatype Parameters - Wrappers

- *Parameterize* any type to a data constructor, using the keyword **of**:

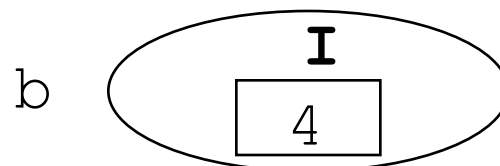
```
type IR = I of int | R of double;;
```

- In effect, such a constructor is a *wrapper* that contains a data item of the given type



```
let a = R 38.5;;
```

```
R 5;      error
```



```
let b = I 4;;
```

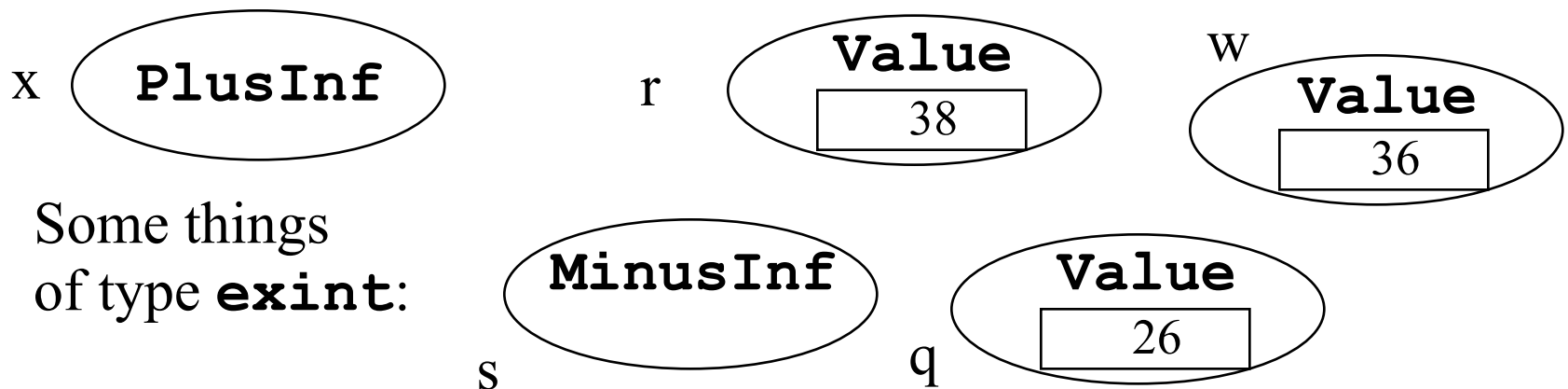
```
I 3.14;  error
```

Wrappers

- You can add a parameter of any type to a data constructor, using the keyword **of**:

```
type exint = Value of int | PlusInf | MinusInf;;
```

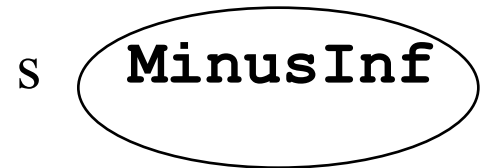
- In effect, such a constructor is a wrapper that contains a data item of the given type



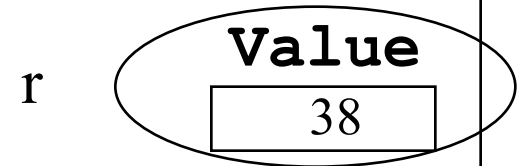
Wrapper Example

```
type exint = Value of int | PlusInf | MinusInf;;  
type exint = MinusInf | PlusInf | Value of int
```

```
let s = MinusInf;;  
val s : exint = MinusInf
```



```
Value;;  
val it : arg0:int -> exint
```



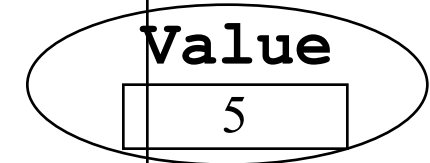
```
let r = Value 38;;  
val r : exint = Value 38
```

- **Value** is a data constructor that takes a parameter: the value of the **int** to store
- Signature is a function that takes an **int** and returns an **exint** containing that **int**

A Value Is **exint** Not **int**

```
let x = Value 5;;  
  val x = Value 5 : exint
```

x



```
x+x;;
```

```
error FS0001: The type 'exint' does  
not support the operator '+'
```

```
(Value 5) + (Value 5);;
```

```
error FS0001: The type 'exint' does  
not support the operator '+'
```

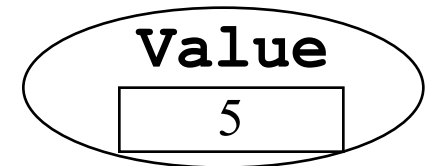
- **Value 5** is an **exint**
- It is not an **int**, though it contains one
- The **int** accessed by pattern matching

Patterns With Data Constructors

```
let x = Value 5;;
```

```
val x : exint = Value 5
```

x

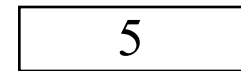


```
let (Value y) = x;;
```

```
Warning: Incomplete pattern matches...
```

```
val y : int = 5
```

y



- To recover a data constructor's parameters, use pattern matching
- So **Value** is no ordinary function: ordinary functions can't be pattern-matched this way

Exhaustive Patterns

```
let x = Value 5;
let s =
  match x with
  | PlusInf -> "infinity"
  | MinusInf -> "-infinity"
  | Value y -> y.ToString();;
val s : string = "5"
```

An **exint** can be a **PlusInf**, a **MinusInf**, or a **Value**

```
let x = Value 5;;
let s =
  match x with
  | PlusInf -> 9999999999999999
  | MinusInf -> -9999999999999999
  | Value y -> y;;
val s : int = 5
```

Pattern-Matching Function

```
let square x =  
  match x with  
  | PlusInf -> PlusInf  
  | MinusInf -> PlusInf  
  | Value x -> Value (x*x) ;;  
val square : x:exint -> exint  
  
square MinusInf;;  
  val it : exint = PlusInf  
  
square (Value 3) ;;  
  val it : exint = Value 9
```

Pattern-matching function definitions are especially important with programmer defined datatypes

Example - extint Factorial

```
let mult (Value x) (Value y) = Value (x*y) ;;  
  val mult : exint -> exint -> exint
```

```
mult (Value 5) (Value 4) ;;  
  val it : exint = Value 20
```

```
let rec fact x =  
  match x with  
  | PlusInf -> PlusInf  
  | MinusInf -> MinusInf  
  | Value 0 -> Value 1  
  | Value n -> mult  
    (fact (Value (n-1))) (Value n) ;;  
  val fact : x:exint -> exint
```

```
fact (Value 5) ;;  
  val it : exint = Value 120
```

Exercise 1

1. Give a datatype declaration for type constructor *color* with data constructor *Red*, *Green*, *Yellow*, *Blue*, *White*.
2. Define function *primary* that returns *true* when the single parameter is red, green or blue; else return *false*.
3. Give a parameterized *type* declaration for type *number* of integer (*AnInt*) or real (*AReal*).
4. `type exint = Value of int | PlusInf | MinusInf;;`

```
let x = AReal 5.0;;  
  val x : number = aReal 5.0  
let y = AnInt 4;;  
  val y : number = anInt 4
```

Exercise 1 Continued

4. Define the function *plus* of type:

`number -> number -> number`

that adds two numbers, coercing *int* to *real* when mixed parameters. Recall *real(3)* converts *int* type to *real*.

```
let x = AReal 5.0;;
let y = AnInt 4;;

plus x y;;
    val it : number = AReal 9.0
plus y y;;
    val it : number = AnInt 8
```

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Type Constructors With Parameters

- Type constructors without parameters:

```
type EXINT = NOINT | SOMEINT of int;;
```

- Type constructors with parameters:

```
type 'a option = NONE | SOME of 'a;;
```

- **'a** is the type parameter; **option** is datatype name
- Type constructor parameters are *type variables*
- Result: a new *polymorphic* type bound at execution time

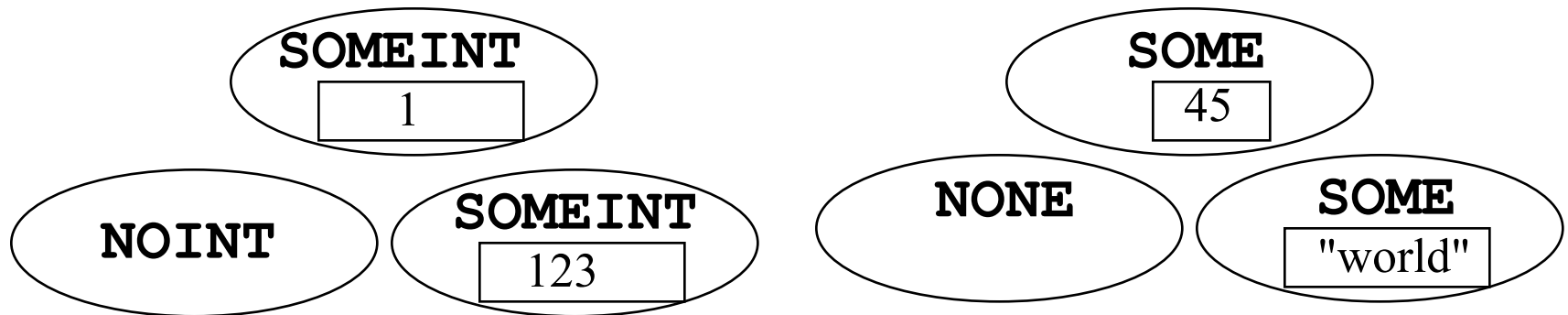
Type Constructors With Parameters

- Type constructors without parameters:

```
type EXINT = NOINT | SOMEINT of int;;
```

- Type constructors with parameters:

```
type 'a option = NONE | SOME of 'a;;
```



Values of type **EXINT**

Values of type
string option and
int option

Parameter Before Name

```
SOME ;;
```

```
val it : arg0:'a -> 'a option
```

```
SOME 4 ;;
```

```
val it : int option = SOME 4
```

```
SOME 1.2 ;;
```

```
val it : float option = SOME 1.2
```

```
SOME "pig" ;;
```

```
val it : string option = SOME "pig"
```

```
SOME [1;2;3] ;
```

```
val it : int list option = SOME [1; 2; 3]
```

- Type constructor parameter **'a** before constructor name:

```
type 'a option = NONE | SOME of 'a ; ;
```
- Types **'a option** and **int option**, just like **'a list** and **int list**.

Uses of **option** datatype

- Predefined type constructor in F#: **None**, **Some**
- Used by predefined functions (or your own) when the result is not always defined

```
let optdiv a b =  
    if b = 0 then NONE else SOME (a / b) ;;  
val optdiv : a:int -> b:int -> int option  
  
optdiv 7 2 ;;  
val it : int option = SOME 3  
  
optdiv 7 0 ;;  
val it : int option = NONE
```


Uses of `option` datatype

```
type 'a option = NONE | SOME of 'a;;
```

```
type exint = Value of int  
           | PlusInf  
           | MinusInf;;
```

```
let x = SOME (Value 3);;
```

```
  val x : exint option = SOME (Value 3)
```

```
SOME (SOME 5);;
```

```
  val it : int option option = SOME (SOME 5)
```

```
SOME (SOME 5, 3.14);;
```

```
  val it : (int option * float) option =  
           SOME (SOME 5, 3.14)
```

Longer Example: **bunch**

```
type 'x bunch = One of 'x
              | Group of 'x list;;
```

- An **'x bunch** is either one of type **'x**, or a list of type **'x**
- As usual, F# infers types:

```
One 1.0;;
```

```
val it : float bunch = One 1.0
```

```
Group [true, false];;
```

```
val it : (bool * bool) bunch =  
Group [(true, false)]
```

Example: Polymorphism

```
type 'x bunch = One of 'x
              | Group of 'x list;;

let size x =
  match x with
  | One _ -> 1
  | Group x -> List.length x;;
  val size : x:'a bunch -> int

size (One 1.0) ;;
  val it : int = 1

size (Group [true; false]) ;;
  val it : int = 2
```

- F# can infer **bunch** types, but does not always have to resolve them, just as with **list** types

Example: No Polymorphism

```
type 'x bunch = One of 'x
              | Group of 'x list;;
let rec sum x =
  match x with
  | One x -> x
  | Group [] -> 0
  | Group (h::t) -> h + (sum (Group t));;
val sum : x:int bunch -> int
sum (One 5) ;;
  val it : int = 5
sum (Group [1,2,3]) ;;
  val it : int = 6
```

- Applied the **+** operator to the list elements
- F# determines **int bunch** the parameter type

```
type bunch = One of int
            | Group of int list;;
```

- a) **One 3;;**
- b) **One "Hi";;**
- c) **Group [1;2;3];;**
- d) **Group ["Hi"; "OK"];;**

Exercise 2

Which are valid?

```
type 'x bunch = One of 'x
               | Group of 'x list;;
```

- e) **One 3;;**
- f) **One "Hi";;**
- g) **Group [1;2;3];;**
- h) **Group ["Hi"; 3];;**
- i) **Group [One 1; One 2];;**
- j) **Group [One "Hi"; One 3];;**
- k) **Group [Group [1;2]; Group[3;4]];;**
- l) **Group [Group ["Hi"; "OK"], Group[1;2]];;**

What is wrong if anything?

Exercise 2

```
type intbunch = One of int
              | Group of int list;;

let rec sum i =
  match i with
  | One i -> i
  | Group [] -> 0
  | Group (h::t) -> (sum (One h)) + (sum (Group t));;
```

Are these the same?

```
type intbunch = One of int
              | Group of int list;;

Group [1;2;3];;

type intbunch = One of int
              | Group of intbunch list;;

Group [One 1; One 2; One 3];;
```

Exercise 2

```
type intnest = INT of int
              | LIST of intnest list;;

let rec intret x =
  match x with
  | INT i -> i
  | LIST [a] -> intret a
  | LIST (h::t) -> intret (LIST t);;

intret (INT 9);;      INT 9      returns 9
intret (LIST [INT 4, INT 3]);; returns 3
                        INT INT
                        LIST 4 3
```

Write function *sum* of type *intnest* that sums all integers in an *intnest*.

Hint: match (INT x) and (LIST L). For (LIST L), L is a list with *hd L* an INT and *tl L* a LIST. Return 0 when L is null, otherwise sum the INT and LIST parts.

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Recursively Defined Type Constructors

The type constructor being defined may be used in its own data constructors:

```
type intlist = INTNIL |  
              INTCONS of int * intlist;;
```

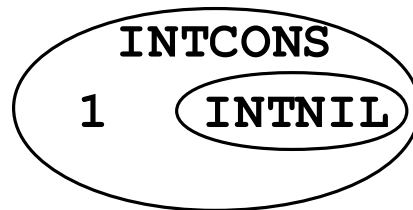
let x = INTNIL;;



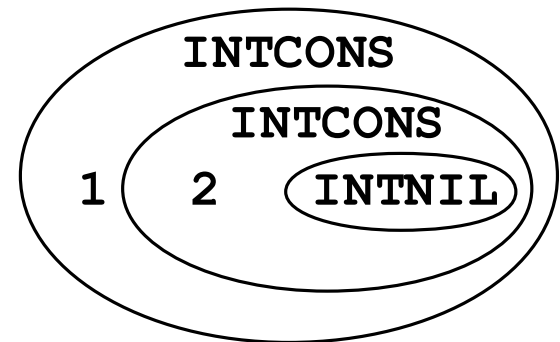
the empty list

Some values of
type **intlist**:

INTCONS(1,INTNIL)



the list [1]



the list [1,2]

INTCONS(1, INTCONS(2,INTNIL))

Construction as a Tuple

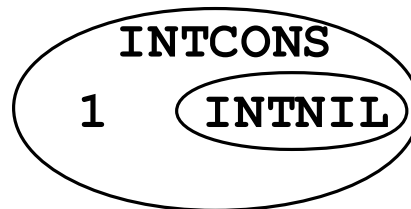
```
INTNIL;;  
val it : intlist = INTNIL  
INTCONS (1,INTNIL) ;;  
val it : intlist = INTCONS (1,INTNIL)  
INTCONS (1, INTCONS (2, INTNIL)) ;;  
val it : intlist = INTCONS (1,INTCONS (2,INTNIL))
```

let x = INTNIL;

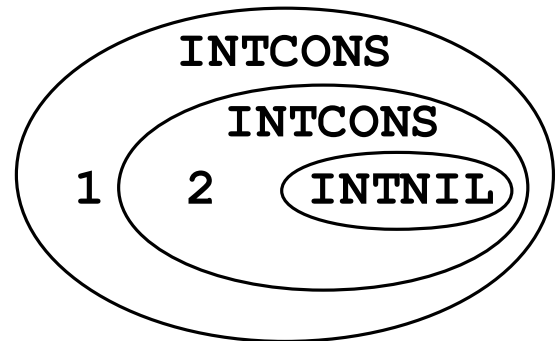


the empty list

INTCONS(1,INTNIL)



the list [1]



the list [1,2]

INTCONS(1, INTCONS(2,INTNIL))

An **intlist** Length Function

```
type intlist = INTNIL |
                INTCONS of int * intlist;;
let rec intlistLength L =
  match L with
  | INTNIL -> 0
  | INTCONS (_, tail) -> 1 +
    (intlistLength tail);;

let x = INTCONS (3, INTCONS (4, INTNIL));

intlistLength x;
  val it : int = 2
```

intlist similar to predefined lists except use of **INTNIL** or **int * intlist** tuples for each element

Parametric List Type

```
type 'element mylist = NIL
| CONS of 'element * 'element mylist;;
```

- A parametric list type using nested tuples, similar to predefined **list**
- **'element** is polymorphic type parameter
- Allows a *mylist* datatype of polymorphic types

```
CONS (1.0, NIL) ;;
val it : float mylist = CONS (1.0, NIL)
CONS (1, CONS (2, NIL)) ;;
val it : int mylist = CONS (1, CONS (2, NIL))
```

Some **mylist** Functions

```
type 'element mylist =  
  | NIL  
  | CONS of 'element * 'element mylist;;
```

```
let rec myLength x =  
  match x with  
  | NIL -> 0  
  | CONS (_, t) -> 1 + (myLength t) ;;  
let rec addup x =  
  match x with  
  | NIL -> 0  
  | CONS (h, t) -> h + (addup t) ;;  
addup (CONS (1, CONS (2, NIL))) ;; returns 3
```

- Similar to predefined **list** type constructor
- Of course, to add up a list could use **foldBack...**

A **foldBack** For **mylist**

```
let rec myFoldBack f c x =  
  match x with  
  | NIL -> c  
  | CONS (a, b) -> f a (myFoldBack f c b) ;;
```

- Definition of a function like **foldBack** that works on '**a mylist**
- Can now add up an **int mylist x** with:
`myFoldBack (fun a b -> a+b) 0 x;;`
- One remaining difference: `::` is an operator and **CONS** is not

Exercise 3

- Write *rev* function to reverse a *mylist*.
- Function *snoc* places an '**e1**' at end of a *mylist*.

```
type 'e1 mylist = NIL
  | CONS of 'e1 * 'e1 mylist;;

let c = CONS (1, CONS (2, CONS (3, NIL))) ;;
val c : int mylist = CONS (1, CONS (2, CONS (3, NIL)))

let rec snoc a x =
  match x with
  | NIL -> CONS (a, NIL)
  | CONS (h, t) -> CONS (h, (snoc a t)) ;;
snoc 4 (CONS (1, CONS (2, NIL))) ;;
val it : int mylist = CONS (1, CONS (2, CONS
(4, NIL)))

rev (CONS (1, CONS (2, NIL))) ;
val it : int mylist = CONS (2, CONS (1, NIL))
```

```

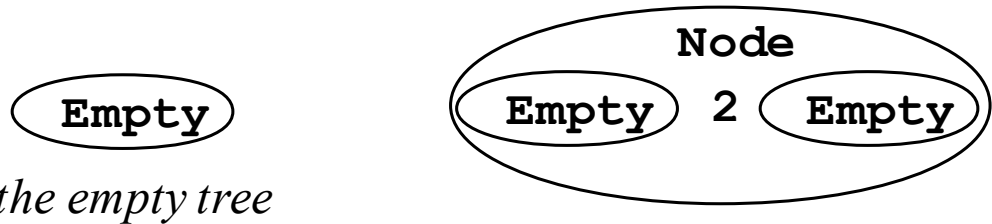
struct tree {
    tree *left, *right;
    char data;
};
void f(tree *t) {
    if(t != NULL) {
        cout << t->data;
        f(t->left);
        f(t->right);
    }
}
void main ( void ) {
    tree  l1={NULL,NULL, '1' },
          l2={NULL,NULL, '2' },
          r2={NULL,NULL, '3' },
          r1={&l2, &r2, '*' },
          t={&l1, &r1, '-' };

    f(&t);
}

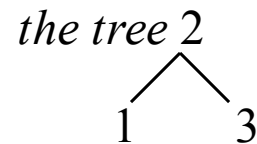
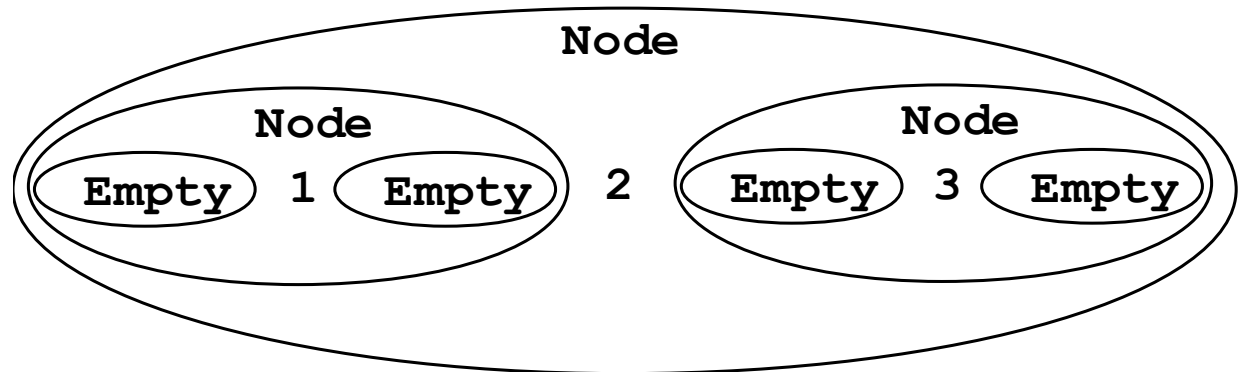
```


Polymorphic Binary Tree

```
type 'data tree =  
  | Empty  
  | Node of 'data tree * 'data * 'data tree;;
```

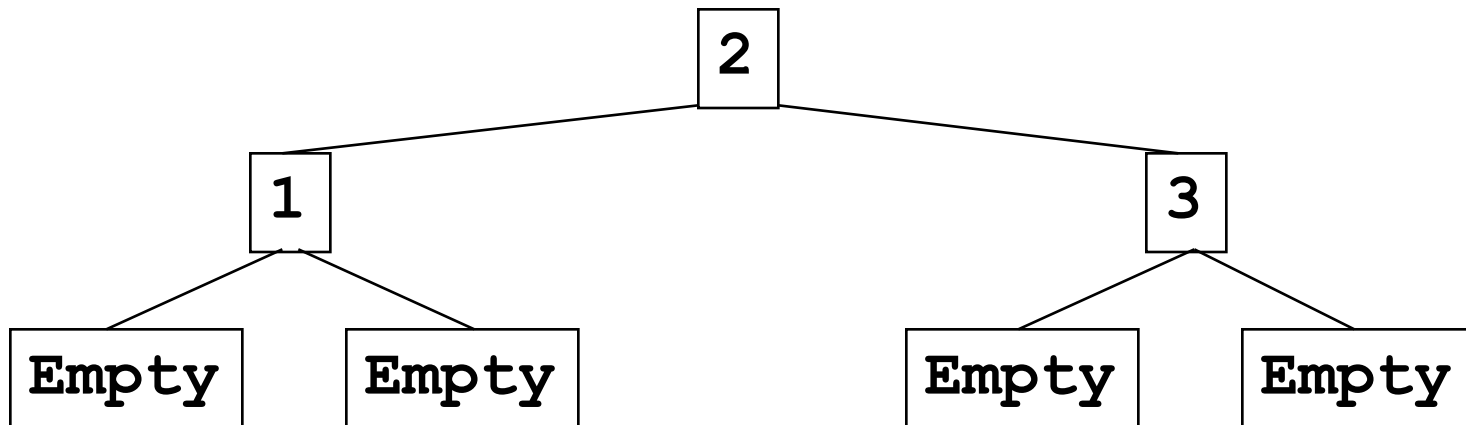


Some values of
type `int tree`:



Constructing Those Values

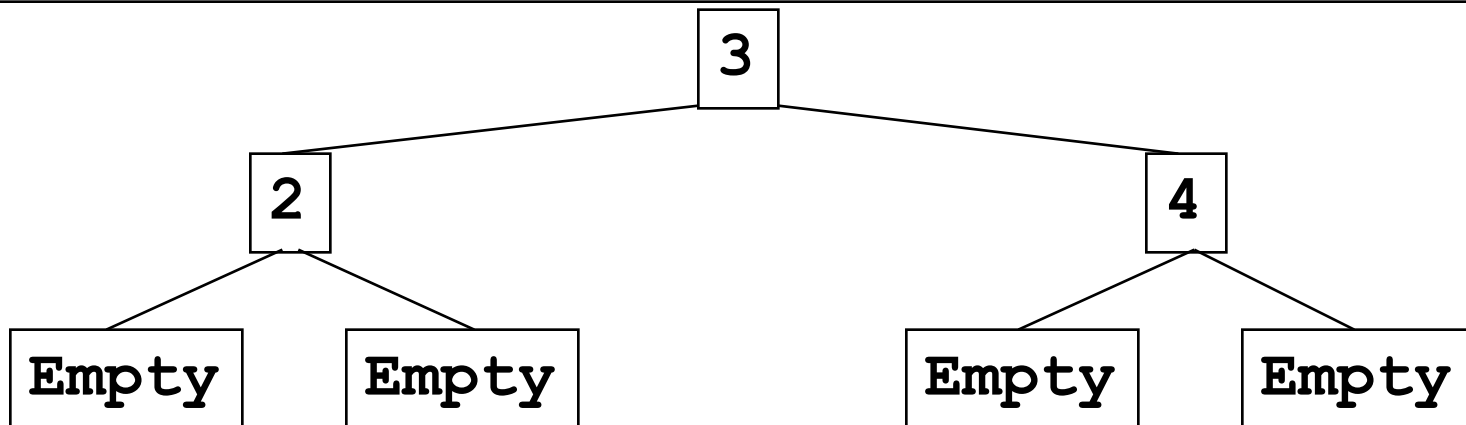
```
let treeEmpty = Empty;;  
  val treeEmpty = Empty : 'a tree  
let tree2 = Node (Empty, 2, Empty);;  
  val tree2 = Node (Empty, 2, Empty) : int tree  
let tree123 = Node (Node (Empty, 1, Empty),  
                  2,  
                  Node (Empty, 3, Empty));;
```



Increment All Elements

```
let rec incall x =  
  match x with  
  | Empty -> Empty  
  | Node (x,y,z) ->  
    Node (incall x, (y+1), (incall z));;
```

```
incall tree123;;  
val it : int tree = Node (Node (Empty,2,Empty),  
  3,  
  Node (Empty,4,Empty))
```



Add Up The Elements

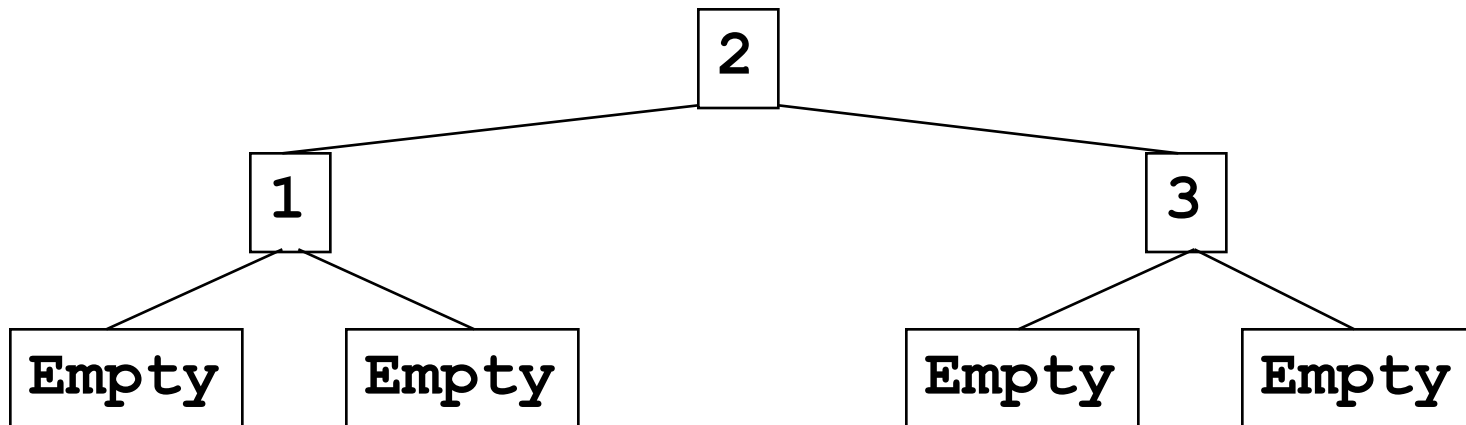
```
let rec sumall x =  
  match x with  
  | Empty -> 0  
  | Node (x,y,z) ->  
    sumall x + y + sumall z;;
```

```
sumall tree123;  
val it : int = 6
```

Convert To List (Polymorphic)

```
let rec listall x =  
  match x with  
  | Empty -> []  
  | Node (x,y,z) ->  
    listall x @ y :: listall z;;
```

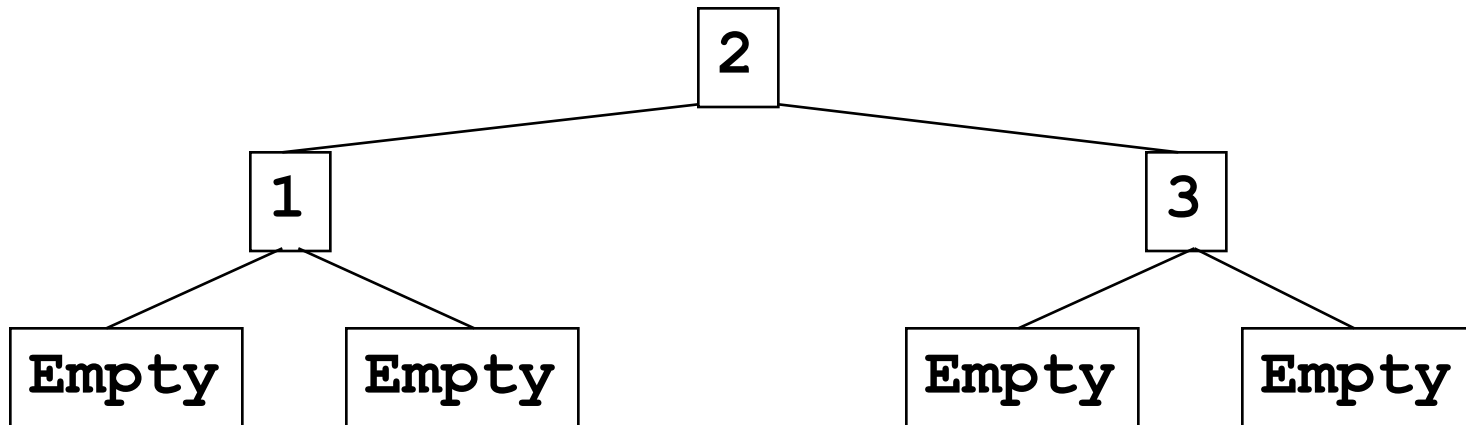
```
listall tree123;;  
val it : int list = [1; 2; 3]
```



Inorder Tree Search

```
let rec isintree x l =  
  match l with  
  | Empty -> false  
  | Node (l,y,r) when x=y -> true  
  | Node (l,y,r) when (isintree x l) -> true  
  | Node (l,y,r) -> isintree x r;;
```

```
isintree 4 tree123;  
val it : bool = false  
isintree 3 tree123;  
val it : bool = true
```



Insert in order

```
let rec insert a x =
  match x with
  | Empty -> Node (Empty, a, Empty)
  | Node (l,d,r) when a < d -> Node(insert a l,d,r)
  | Node (l,d,r) -> Node(l,d,insert a r);;

let t = insert 5 Empty;;
let t = insert 4 t;;
let t = insert 7 t;;
let t = insert 6 t;;
val t = Node (
  Node (Empty, 4, Empty) ,
  5,
  Node (
    Node (Empty, 6, Empty) ,
    7,
    Empty) )
```

Exercise 4

- Write function *inorder* to return a string consisting of the concatenation of tree node data values when traversed in-order.
- Modify **sumall** function in minor way.
- Use *5.ToString()* to convert integer 5 to a string.
- “a” + “b” + “c” concatenates to “abc”.

```
inorder Node(Node(Empty,4,Empty),5,Node(Node(Empty,6,Empty),7,Empty));;  
val it : string = "4567"
```

```
let sumall x =  
    match x with  
    | Empty -> 0  
    | Node (x,y,z) -> sumall x + y + (sumall z) ;;
```


Outline

- Enumerations
- Data constructors with parameters
- Type constructors with parameters
- Recursively defined type constructors
- **Farewell to F#**

That's All For Now

- That's all the F# we will see for a while
- There is, of course, a lot more
- A few words about the parts we skipped:
 - records (like tuples with named fields)
 - arrays, with elements that can be altered
 - references, for values that can be altered
 - exception handling in depth

More Parts We Skipped

- support for encapsulation and data hiding:
 - structures: collections of datatypes, functions, etc.
 - signatures: interfaces for structures
 - functors: like functions that operate on structures, allowing type variables and other things to be instantiated across a whole structure

More Parts We Skipped

- API: the standard basis
 - predefined functions, types, etc.
 - Some at the top level but most in structures:
String.replicate, **cos** operator,
List.nth, etc.

More Parts We Skipped

- F# pipelines
- Modules for organizing code
- Object oriented F#

- Other dialects besides F#
 - OCaml
 - Standard ML (SML)
 - Concurrent ML (CML) extensions

Functional Languages

- F# supports a function-oriented style of programming
- If you like that style, there are many other languages to explore, like Lisp and Haskell